RENORMALIZATION FLOW OF THE GRIMUS-NEUFELD MODEL AT ONE LOOP LEVEL

Rokas Garbačauskas[1], Thomas Gajdosik[1]

[1]Vilnius university, institute of theoretical physics and astronomy, Lithuania
rokas.garbacauskas@ff.stud.vu.lt

The Standard Model (SM) of particle physics is a very succesful theory. Taking as an input 19 (or 26 if one includes neutrino masses and their mixing angles) input parameters it produces a lot of testable predictions which up to now have withstood the experimental tests almost flawlessly. In fact some of the theoretical groundwork helped to make interesting discoveries, latest of which is the discovery of the Higgs boson.

On the other hand, the current widely accepted formulation of the SM is incomplete. For example the mechanism generating the aforementioned neutrino masses is still an open question. It has been experimentally verified in many independent experiments [1] that neutrinos can change their gauge eigenstates. This phenomenon is called neutrino oscillation. The mathematical apparatus that describes this requires that neutrinos have a mass. Many different SM additions have been proposed to accomplish this and are waiting for more experimental data to narrow down the parameter space. One such model is the Grimus-Neuefeld model (GNM) [2] which attempts to describe the two measured neutrino mass differences by an addition of an extra Higgs doublet and a sterile Majorana neutrino which via radiative mass corrections at 1 loop and the seesaw mechanism respectively do just that.

In our study we were searching for analytical expressions of the beta functions

$$\beta_\alpha = \mu \frac{d}{d\mu}\alpha. \tag{1}$$

which describe the change of the given parameters with respect to scale. Calculations involved corrections up to one loop level for some parameters essential for GNM, namely the gauge couplings of U(1) and SU(2), Yukawa couplings and the mass of the Majorana neutrino.

Achieving this required writing down the Lagrangian densities ($\mathscr{L}$) of these sectors in D dimensions in a consistent way. That is in order for the action ($S = \int d^D x \mathscr{L}$) to be dimensionless all terms of $\mathscr{L}$ have to be of mass dimension D, i. e. $[\mathscr{L}] = D$ (as usual in particle physics $\hbar = c = 1$). It is then conventional to rescale the couplings of the theory with a parameter μ ($[\mu] = 1$) in a way that the coupling itself remains a dimensionless number. For examle, in QCD $e \to \mu^{\frac{4-D}{2}} e$.

Feynman formalism was then used to write down amplitudes for events described by the parameters of interest. These amplitudes suffer from one of the issues that slowed down the advance of the mathematical apparatus behind the SM - Quantum Field Theory (QFT) - in mid XXth century: infinities emerging when evaluating integrals of some loop level corrections. The integrals generally have the form of

$$T^N_{\mu_1 \dots \mu_P}(p_1, \dots, p_{N-1}, m_0, \dots, m_{N-1}) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int d^D q \frac{q_{\mu 1} \cdots q_{\mu P}}{D_0 D_1 \cdots D_{N-1}}, \tag{2}$$

where D is the dimension number, N is the number of propagator factors, P is the number of the integration momenta in the numerator and

$$D_i = (q + p_i)^2 - m_i^2 + i\varepsilon, \; p_0 = 0 \text{ and } i = 1, \dots, N-1.$$

In general these integrals diverge for $D \geq 4$ [3].

These divergences spurred the development of an entire new branch of mathematics called regularization. There are many regularization schemes but we used the most popular one - dimensional regularization - as it preserves Lorentz invariance. This approach lets the dimensionality of the problem stray from 4 by a small amount ε: $D = 4 - \varepsilon$. This allows to get analytical expression for the given integral. The divergent part is easily isolated and is written down in a counterterm Δ_α along with some constants depending on the convention and the dependance on the scale parameter. The couplings responsible for this specific interaction or field values are shifted by this counterterm. This allows to get finite values for measurable quantities as the diverging parts simply cancel out so ε can be safely sent to 0 at the end of the calculations. This also introduces the notion of running couplings as the new renormalized couplings are scale dependent. And details of this dependance are described by the beta functions eq. (1).

[1] M. G. Aartsen, M. Ackermann, J. Adams, et al., Search for nonstandard neutrino interactions with icecube deepcore, Phys. Rev. D, Apr 2018, 97, 072009

[2] Draukšas, Simonas & Dūdėnas, Vytautas & Gajdosik, Thomas & Juodagalvis, Andrius & Juodsnukis, Paulius & Jurčiukonis, Darius. (2019). The Grimus–Neufeld Model with FlexibleSUSY at One-Loop. Symmetry. 11. 1418. 10.3390/sym11111418.

[3] A. Denner, Techniques for the calculation of electroweak radiative corrections at the one-loop level and results for w-physics at lep200, 2007.