MATHEMATICAL MODELING OF COATING BASED ACHROMATIC WAVEPLATE

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The discovery and understanding of circular polarization vision in a mantis shrimp (*Stomatopod Crustacean)* [1] inspired the creation of an achromatic optical coating based quarter-wave retarder [2]. Achromatic phase retardation is exceptionally unusual optical phenomena due to the dispersion of materials. Achromatic phase retardation can be obtained in thin film optical coatings made of birefringent materials where the refractive index is different for x and y components of e-vector and it also depends on the wavelength of incident light. Thin film optical coating based waveplate is a layered structure which properties depend on multiple parameters, such as number and thickness of layers, refraction index and wavelength of light. Since the desired waveplate is achromatic, the dependence on wavelength of light must be eliminated. Selection of the correct parameters can not be done analytically but the system can be designed and parameters found by using mathematical models and numerical optimization methods.

Phase retardation can be calculated from [3]:

$$\xi = \arctan\left[\frac{-\mathrm{Im}(\eta_0 B + C)}{\mathrm{Re}(\eta_0 B + C)}\right] \tag{1}$$

Where η_0 is the admittance of incident medium and *B, C* are the parameters of thin film optical coating:

$$\begin{bmatrix} B \\ C \end{bmatrix} = \left(\prod_{r=1}^{q} \begin{bmatrix} \cos\delta_\mathrm{r} & (i\sin\delta_\mathrm{r}/\eta_\mathrm{r} \\ i\eta_\mathrm{r}\sin\delta_\mathrm{r} & \cos\delta_\mathrm{r} \end{bmatrix}\right) \begin{bmatrix} 1 \\ \eta_\mathrm{m} \end{bmatrix} \tag{2}$$

Where the characteristic matrix of a single layer is multiplied q times, q is the number of layers in thin film optical coating. η_r is the admittance of incident medium, η_m is the admittance of substrate, phase thickness of a coating $\delta_\mathrm{r} = 2\pi n d \cos\alpha/\lambda$ where *n* is refractive index, *d* is physical thickness of a layer, α is incident angle.

But according to Herpin's theorem a symmentrical thin-film combination ABA is equivalent to a single film [4] and therefore its phase retardation can be calculated in a simple way:

$$\xi = \mathrm{q}(\gamma_\mathrm{y} - \gamma_\mathrm{x}) \tag{3}$$

Where phase thickness of a coating for one polarization:

$$\gamma = \arccos\left(\cos 2\delta_\mathrm{A}\cos\delta_\mathrm{B} - \frac{1}{2}\left(\frac{n^\mathrm{B}}{n^\mathrm{A}} - \frac{n^\mathrm{A}}{n^\mathrm{B}}\right)\sin 2\delta_\mathrm{A}\sin 2\delta_\mathrm{B}\right) \tag{4}$$

There is no need to multiply matrices q times and when more layers are added to have higher phase retardation, the multiplication of one symmetrical layer phase retardation by q is done.

By calculating all possible phase retardation values for different wavelenghts, refractive index values and physical thickness values, the best physical thickness of layers can be found by data analysis, comparing deviation and mean values of each data set. Choosing data set with lowest deviation (when phase retardation values distribute uniformly throughout the spectra and therefore waveplate is achromatic) and highest mean (desired phase retardation value can be obtained by adding less layers) best physical thickness values for achromatic waveplate can be found.

Also, using Nelder-Mead simplex algorithm lead to the same results and reduces computation time. The minima of function, where difference of phase retardation with given parameters and desired phase retardation is mathematically described, gives values of physical thickness (when refractive index values are fixed) which can be used to produce achromatic optical coating waveplate.

All aforementioned methods have been tested to create the model of an achromatic waveplate in visible regime with phase retardation equal to $\pi/2$. Several designs have been found and compared. Optimal values of refractive index and physical thickness have been computed.

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